Lecture 1 «Basics of heat exchange. Temperature field. Temperature gradient. Fundamental law of heat conductivity. Differential equation of heat conductivity. Heat conductivity of a flat wall at a stationary mode. Heat conductivity of a cylindrical wall»

Aim: Give the basics of heat exchange. Characterize the temperature field. Formulate the temperature gradient. Describe the fundamental law of heat conductivity. Write the differential equation of heat conductivity. Describe the heat conductivity of a flat wall at a stationary mode. Characterize the heat conductivity of a cylindrical wall.

Lecture summary: ***Basics of heat exchange.*** The transfer of energy in the form of heat, occurring between bodies having different temperatures, is called *heat exchange*. The driving force of any transfer of heat transfer is the difference in temperatures of the more heated and less heated bodies, in the presence of which heat spontaneously, in accordance with the second law of thermodynamics, passes from a warmer to a less heated body. Heat exchange between bodies is the exchange of energy between molecules, atoms and free electrons: as a result of heat exchange, the intensity of the motion of particles of a more heated body decreases, and less heated – increases. The bodies involved in heat exchange are called *heat carriers*.

Heat transfer – the science of heat transfer processes. The laws of heat transfer form the basis of thermal processes – heating, cooling, condensation of vapor, evaporation – and are of great importance for carrying out many mass-exchange processes (processes of distillation, drying, etc.), as well as chemical processes that occur with the supply or removal of heat.

There are three ways of spreading heat: *thermal (heat) conductivity*, *convection* and *radiation*.

Thermal (heat) conductivity refers to the process of transfer of thermal energy by direct contact between particles of a body with different temperatures.

Convection refers to the process of heat transfer by moving and mixing liquid or gas particles together. The transfer of heat by convection is always accompanied by thermal conductivity, since direct contact of particles with different temperatures is carried out. Simultaneous heat transfer by convection and thermal conductivity is called convective heat transfer; it can be free and forced.

If the movement of the body is caused artificially (by a compressor, a fan, a mixer, etc.), then such convective heat transfer is called *forced*. If the motion of the working fluid arises under the influence of the difference in the densities of the individual parts of the liquid from heating, then such heat exchange is called *free*.

Radiation (radiation) is the process of energy transfer in the form of electromagnetic waves. All three methods of heat exchange arise in the presence of a temperature difference of individual parts of the body, or several bodies. This temperature difference is the driving force under the action of which there is a transfer of heat.

Temperature field. Temperature gradient

Among the basic problems of the theory of heat transfer is the establishment of a relationship between the heat flow and the distribution of temperatures in the environments. The set of instantaneous values of any quantity at all points of a given environment (body) is called the field of this quantity. Accordingly, the set of temperature values at a given instant of time for all points of the environment is called a temperature field.

In the most general case, the temperature at a given point t depends on the coordinates of the point (x, y, z) and varies with a time τ, i.e. the temperature field is expressed by a function of the form:

$$t = f(x, y, z, \tau) \tag{1}$$

This dependence is the equation of an unsteady (non-stationary) temperature field.

If the temperature of the body is a function of only the coordinates and does not change with time, the temperature field will be stationary (steady-state).

$$t = f(x, y, z); \quad \frac{\partial t}{\partial \tau} = 0. \tag{2}$$

Unlike the temperature, which is a scalar quantity, the heat flow associated with the direction of heat transfer is a vector value.

The temperature in the body can vary in the direction of one, two and three coordinate axes. In accordance with this, the temperature field can be one-, two-, and three-dimensional.

One-dimensional, for example, is the problem of heat transfer in a wall, in which the length and width are infinitely large in comparison with thickness. For this case, the equation of the temperature field for the nonstationary regime

$$t = f(x, \tau); \quad \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0. \tag{3}$$

For stationary mode

$$t = f(x); \ \frac{\partial t}{\partial \tau} = 0 \ u \ \frac{\partial t}{\partial y} = \frac{\partial t}{\partial z} = 0. \tag{4}$$

For any temperature field in the body, there are always points with the same temperature. If we connect the points of the body with the same temperature, we obtain isothermal surfaces that never intersect each other. They either close to themselves, or end at the boundaries of the body. Consequently, the temperature in the body changes only in the direction crossing the isotherms.

Let that the temperature difference between two nearby isothermal surfaces is *Δt* (Fig. 1). The shortest distance between these surfaces is the distance along the normal *Δn*. On approaching these surfaces, the deviation *Δt/Δn* tends to the limit

$$\lim \left(\frac{\Delta \mathrm{t}}{\Delta \mathrm{n}}\right)_{\Delta \mathrm{n} \to 0} = \frac{\partial \mathrm{t}}{\partial \mathrm{n}} = \mathrm{grad\ t} \tag{5}$$

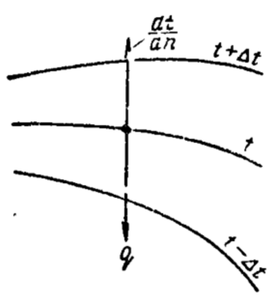

Fig. 1. To the determination of the temperature gradient to the expression of the Fourier's law

The derivative of the temperature along the normal to the isothermal surface is called *the temperature gradient*. This gradient is a vector whose direction corresponds to an increase in temperature. The heat transfer takes place along the line of the temperature gradient, but is directed in the direction opposite to this gradient: *q* ~ *(dt/dn)*.

Fundamental law of heat conductivity

The condition for the transfer of heat through thermal conductivity is the presence of a temperature difference at various points in the body.

The basic law of heat transfer by thermal conductivity is the Fourier's law, according to which the amount of heat *dQ* transmitted through thermal conductivity through a surface element *dF* perpendicular to the heat flow during time $d\tau$ is directly proportional to the temperature gradient, surface and time:

$$dQ = -\lambda \frac{\partial t}{\partial n} dF d\tau \qquad (6)$$

or the amount of heat transmitted per unit surface per unit time

$$q = \frac{Q}{F\tau} = -\lambda \frac{\partial t}{\partial n} \qquad (7)$$

The quantity *q* is called *the heat flow density*.

The minus sign in front of the right-hand side of equations (6) and (7) indicates that the heat moves toward a temperature drop.

The coefficient of proportionality λ in equations (6) and (7) is called *the coefficient of thermal conductivity*. It characterizes the ability of a substance to conduct heat. The dimension λ is found from equation (6)

$$[\lambda] = \left[\frac{dQ\partial n}{dF\partial\tau}\right] = \left[\frac{J\cdot m}{m^2\cdot s\cdot K}\right] = \left[\frac{W}{m\cdot K}\right] \qquad (8)$$

When *Q* is expressed in *kcal/h*

$$[\lambda] = \left[\frac{dQ\partial n}{dF\partial\tau}\right] = \left[\frac{kcal\cdot m}{m^2\cdot h\cdot K}\right] = \left[\frac{kcal}{m\cdot h\cdot K}\right] \qquad (9)$$

Thus, the coefficient of thermal conductivity shows how much heat passes through heat conduction per unit of time through a unit of the heat exchange surface when the temperature falls by 1 degree per unit length of the normal to the isothermal surface.

The value λ, which characterizes the ability of a body to conduct heat by thermal conductivity, depends on the nature of the substance, its structure, temperature and some other factors.

Differential equation of heat conductivity

From the equation (10) distribution of temperature can be determined only for bodies of a simple configuration – a plate, pipes. Generally this distribution can be received only as a result of the solution of the special differential equation of heat conductivity. This equation is removed on the basis of the energy conservation law with using of a method of mathematical physics. Without a conclusion:

$$\frac{\partial\theta}{\partial\tau} = \frac{\lambda}{c_p\rho}\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) = a\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2}\right) \tag{10}$$

where and = λ/cρ, is called as coefficient of heat diffusivity, the m^2/s, equal to the relation of coefficient of heat conductivity to a volume specific thermal capacity of substance and is a measure of speed of equalization of a temperature field.

It characterizes heat inertial properties of a body: other things being equal that body which possesses big coefficient of heat diffusivity heats quicker up or cooled. The equation (2) describing spatial and temporary change of temperature, belongs to unsteady processes of heat conductivity. For the established processes of $d\theta/d\tau = 0$, and the equation of heat conductivity assumes then simpler look:

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial z^2} = 0 \tag{11}$$

The equations (10) and (11) assume simultaneous change of body temperatures in the directions of all three axes of coordinates therefore them often call the equations of three-dimensional temperature fields.

The equation of heat conductivity of a flat wall

Let us consider the transfer of heat by thermal conductivity through a plane wall of thickness δ with the coefficient of thermal conductivity of the wall material λ (Fig. 2). The temperature changes only in the direction of the x axis. The temperature on the external surfaces is maintained at a constant t_{w_1} and t_{w_2}.

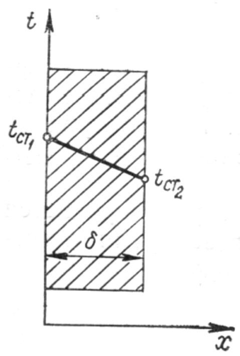

Fig. 2. To the derivation of the equation of the thermal conductivity of a flat wall

Under these conditions, the amount of heat that is transferred by thermal conductivity through the wall surface F in time τ, according to the Fourier's law:

$$Q = -\lambda F \frac{dt}{dx} \quad (12)$$

Dividing the variables, we get

$$dt = -\frac{Q}{\lambda F} dx \quad (13)$$

Integrating equation (13) with the condition Q = *const*, we find

$$t = -\frac{Q}{\lambda F} x + c \quad (14)$$

The integration constant c is determined from the boundary conditions:
at $x = 0$, $t = t_{w_1} = c$;
at $x = \delta$, $t = t_{w_2} = -\frac{Q}{\lambda F}\delta + t_{w_1}$, whence

$$Q = \frac{\lambda}{\delta} F\left(t_{w_1} - t_{w_2}\right), \text{W} \quad (15)$$

Equation (15) can be used to calculate and in this form:

$$Q = \frac{\lambda}{\delta} F\left(t_{w_1} - t_{w_2}\right)\tau, \text{J (kcal)} \quad (16)$$

The calculated heat conductivity formula for the steady heat flow through a multilayer flat wall is derived from the heat conductivity equation for individual layers. In general, the equation has the form:

$$Q = \frac{(t_{w_1} - t_{w_2})F}{\frac{\delta_1}{\lambda_1} + \frac{\delta_2}{\lambda_2} + \cdots + \frac{\delta_n}{\lambda_n}}, \text{W} \quad (17)$$

where δ_1 – the thickness of the first layer, δ_2 – the thickness of the second layer, and δ_n – the thickness of the n^{th} layer of the wall; respectively, the coefficients of thermal conductivity of the layers are equal to $\lambda_1, \lambda_2, \cdots, \lambda_n$; t_{w_1} и t_{w_2} – the temperature of the outer surfaces of the wall.

Equation of heat conductivity of a cylindrical wall

Consider a homogeneous cylindrical wall of length l with inner diameter d_1 and outer diameter d_2. The coefficient of thermal conductivity of the material is constant and is equal to λ. The internal temperature t_1 and external temperature t_2 are kept constant, and $t_1 > t_2$ (Fig. 3). The temperature changes only in the radial direction.

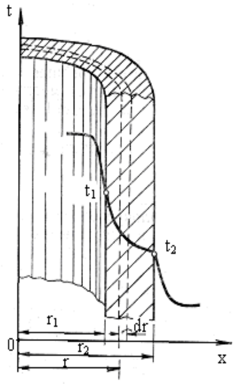

Fig. 3. To the derivation of equation of the heat conductivity of a single-layer cylindrical wall

We select in the wall a ring layer with a radius *r* and thickness *dr*. According to the Fourier's law, the amount of heat passing through such a layer is equal to

$$Q = -\lambda F \frac{dt}{dr} = -\lambda 2\pi r l \frac{dt}{dr} \qquad (18)$$

Dividing the variables, we get

$$dt = -\frac{Q}{2\pi\lambda l} \cdot \frac{dr}{r} \qquad (19)$$

Integrating equation (19) in the range from t_1 to t_2 and from r_1 and r_2, (for λ = const), we obtain

$$\int_{t_1}^{t_2} dt = -\int_{r_1}^{r_2} \frac{Q}{2\pi\lambda l} \cdot \frac{dr}{r} \qquad (20)$$

or

$$t_1 - t_2 = \frac{Q}{2\pi\lambda l} \cdot ln\frac{r_2}{r_1}, \qquad (21)$$

whence

$$Q = \frac{l(t_1 - t_2)}{\frac{1}{2\pi\lambda} ln\frac{d_2}{d_1}}, \text{ W} \qquad (22)$$

Expression (23) is the equation of the thermal conductivity of a homogeneous cylindrical wall for a steady heat flow.

By analogy with the conclusion given for a single-layer wall, for a cylindrical wall consisting of *n* layers, the amount of heat transferred by thermal conductivity is

$$Q = \frac{2\pi l(t_1 - t_2)}{\frac{1}{\lambda_1} ln\frac{d_2}{d_1} + \frac{1}{\lambda_2} ln\frac{d_3}{d_2} + \frac{1}{\lambda_3} ln\frac{d_4}{d_3} + \cdots}, \qquad (23)$$

where d_1 and d_2, d_2 and d_3, d_3 and d_4, etc. – inner and outer diameters of each cylindrical layer.

Questions to control:

1. What types of heat transfer are involved in heat exchange?
2. What is the heat balance in the heat exchangers?
3. Give the notion of an isothermal surface.
4. What is the temperature field?
5. Define the temperature gradient.
6. Give the Fourier's law and explain the physical meaning of the coefficient of thermal conductivity.
7. Derive the heat equation for a single flat wall.
8. What is the thermal resistance of the wall? What is its dimension?
9. What is the reason for the different temperature distribution over the thicknesses of the flat and cylindrical walls?
10. Derive the heat equation for a cylindrical wall.

Literature

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